$\therefore$ of R, $\{-[\frac{1}{2}(s-a)+\frac{1}{2}(s-c)\cos B],\ -\frac{1}{2}(s-c)\sin B\}$;

of Q, $\{\frac{1}{2}[s+a+(s-b)\cos C],\ -\frac{1}{2}(s-b)\sin C\}$;

of P, $\{\frac{1}{2}[s(\cos C+\cos B)-a],\ \frac{1}{2}s(\sin C+\sin B\}$.

$BH=r\cot\frac{1}{2}B$, $OH=r$, $bE=x$, $BE=x\cot\frac{1}{2}B$.

$\therefore\ (r-x)^2\cot^2\frac{1}{2}B+(r-x)^2=(r+x)^2$. $\therefore\ x=r(1-\sin\frac{1}{2}B)/(1+\sin\frac{1}{2}B)$.

Let $r(1-\sin\frac{1}{2}B)/(1+\sin\frac{1}{2}B)=m$, $r(1-\sin\frac{1}{2}C)/(1+\sin\frac{1}{2}C)=n$, $r(1-\sin\frac{1}{2}A)/(1+\sin\frac{1}{2}A)=l$.

$\therefore$ coördinates of b are $(m\cot\frac{1}{2}B,\ m)$; of c, $(a-n\cot\frac{1}{2}C,\ n)$; of a, $(p\cos aBC,\ p\sin aBC)$, where $p=\sqrt{[c^2-2cl\cot\frac{1}{2}B+l^2\operatorname{cosec}^2\frac{1}{2}A]}$, and $\tan aAB=(c\tan B-l\tan B\cot\frac{1}{2}A-l)/(c-l\cot\frac{1}{2}A+l\tan B)$.

Substituting in (1) and (2) the truth of the proposition appears.

By substituting the coördinates of P, Q, R, and a, b, c in (1), (2) we get, after a prodigious amount of work, the coördinates of two points. If the line through these two points coincides with the line through O, M, the proposition is true.

[NOTE.—Dr. Zerr furnished a very beautiful figure to go with his solution, but we lacked the time to engrave it. EDITOR.]

83. Proposed by **WILLIAM HOOVER, A. M., Ph. D.,** Professor of Mathematics and Astronomy in Ohio University, Athens, O.

θ being variable, find the locus of a point whose coördinates are

$$a\tan(\theta+\alpha),\quad b\tan(\theta+\beta).$$

Solution by the PROPOSER.

The rectilinear coördinates being x and y, $x=a\tan(\theta+\alpha)$..........(1), $y=b\tan(\theta+\beta)$.............(2). (1) gives $\theta+\alpha=\tan^{-1}(x/a)$..............(3), $\theta+\beta=\tan^{-1}(y/b)$....(4). Eliminating θ, $\tan^{-1}(x/a)-\tan^{-1}(y/b)=\alpha-\beta$....(5).

Taking tangents of both members of (5) and reducing,

$$xy-\cot(\alpha-\beta)(bx-ay)+ab=0\ldots\ldots\ldots\ldots\ldots\ldots(6),$$

the equation to the required locus.

Solved in a similar manner by *COOPER D. SCHMITT*, *T. W. PALMER*, *OTTO CLAYTON*, and *G. B. M. ZERR*.

CALCULUS.

65. Proposed by **GEORGE LILLEY, Ph. D., LL. D.,** Professor of Mathematics, State University, Eugene, Ore.

A string is wound spirally 100 times around a cone 100 feet high and 2 feet in diameter at the base. Through what distance will a duck swim in unwinding the string keeping it taut at all times, the cone standing on its base and at right angles to the surface of the water ?